COURSE DETAILS

"GEOMETRIA E ALGEBRA"

SSD MAT/03

DEGREE PROGRAMME: BACHELOR DEGREE IN COMPUTER ENGINEERING

ACADEMIC YEAR: 2023-2024

GENERAL INFORMATION – TEACHER REFERENCES

TEACHER: MULTIPLE STUDY COURSE
PHONE:
EMAIL:

SEE THE STUDY COURSE WEBSITE

GENERAL INFORMATION ABOUT THE COURSE

INTEGRATED COURSE (IF APPLICABLE): N.A.
MODULE (IF APPLICABLE): N.A.
CHANNEL (IF APPLICABLE): N.A.
YEAR OF THE DEGREE PROGRAMME (I, II, III): I
SEMESTER (I, II): II
CFU: 6

REQUIRED PRELIMINARY COURSES (IF MENTIONED IN THE COURSE STRUCTURE "REGOLAMENTO")

None.

PREREQUISITES (IF APPLICABLE)

The mathematical content of secondary school curricula.

LEARNING GOALS

Students will acquire the basic tools of linear algebra and geometry. The aim of this course is, on the one hand, to accustom the student to face formal problems, using appropriate tools and correct language, and on the other to solve specific problems of algebraic and geometric type, with the classical tools of linear algebra.

EXPECTED LEARNING OUTCOMES (DUBLIN DESCRIPTORS)

Knowledge and understanding

Lo studente deve dimostrare di conoscere le nozioni (definizioni, enunciati, dimostrazioni se previste dal programma) relative alle strutture algebriche e geometriche studiate (spazi vettoriali, spazi della geometria elementare in dimensione 2 e 3, spazi di matrici) e gli strumenti di calcolo sviluppati, e saper comprendere argomenti affini elaborando le nozioni acquisite.

Applying knowledge and understanding

Lo studente deve dimostrare di saper applicare quanto appreso nella risoluzione di esercizi di verifica elaborati dal Docente, in linea di massima legati ad argomenti quali : rette e piani, matrici, equazioni, vettori. Lo studente deve, inoltre, dimostrare di conoscere le problematiche relative alle strutture algebriche.

COURSE CONTENT/SYLLABUS

References to set theory and algebraic structures: 0,5 CFU

Union, intersection, complement, Cartesian product; correspondences and relationships, applications or functions, restrictions, injective, surjective, bijective, application composition, characterization of objective applications; equivalence relations (example: equivalence between applied vectors). Internal operations: associative property, existence of the neutral element (and uniqueness), existence of symmetric elements (and uniqueness, if the operation satisfies the associative property), commutative property, (examples: addition operations in numerical sets and on free and applied vectors). Abelian and non-abelian groups (examples). Definition of field. Examples: real numbers field, field whose support contains only two elements. External operations (example: external multiplication operation on free and applied vectors.

Vector and Euclidean spaces (over one field): 1,5 CFU

Definition, elementary properties; examples (vector spaces of numbers, polynomials, matrices, free and applied vectors of elementary geometry). Linear combinations, dependence and linear independence and their characterizations; generator systems. Vector subspaces and characterization; sets of vectors that generate the same vector subspace; bases and components of a carrier in an ordered basis; theorem of extraction of a base from a system of generators; Steinitz's lemma and consequences: dimension of a vector space, completion theorem in a basis of a linearly independent set; intersection subspace, sum subspace, direct sum, Grassmann relation. Euclidean vector spaces: scalar product in a vector space on the reals: length of a vector, angle between two vectors, existence of orthonormal bases: Gram-Schmidt procedure; Canonical (or natural) scalar product between numerical vectors. Scalar product between geometric vectors. Calculation of a scalar product using the components of vectors in an ordered orthonormal basis. Pythagorean theorem.

Matrices and determinants: 1 CFU

Elementary line operations; matrices reduced to steps. The rank of a matrix and the number of pivots of a stepped matrix. Triangular and diagonal matrices; product rows by columns; classical definition of determinant (with the use of permutations) and elementary properties (without proof); characterization of the maximum rank by not cancelling the determinant; methods of calculating the determinant: statements of Laplace's Theorem and Laplace's second theorem; statement of the Edged Theorem (Kronecker); invertible matrices and determination of the inverse matrix; similar matrices.

Linear systems: 1 CFU
Solutions, compatibility (Rouchè-Capelli theorem); Cramer's theorem; stepwise method (Gaussian elimination method) and solving a system of linear equations; determination of a basis of the vector space of solutions of a homogeneous linear system; Every subspace of a vector number space is the solution space of a homogeneous linear system and vice versa: Cartesian and parametric representation of the numerical vector subspaces.

Linear applications: 0.5 CFU
Definition and first properties; conservation of linear dependence; core and image; characterization of linear injective and surjective applications; fundamental theorem of linear applications; endomorphisms, isomorphisms; isomorphism associated with an ordered basis; Associated and basic change matrices. Statement of the Size Theorem. Similarity relationship between matrices associated with endomorphisms in different ordered bases.

Diagonalization of endomorphisms and matrices: 0,5 CFU
Eigenvalues, eigenvectors and eigenspaces of endomorphisms (and square matrices); characteristic polynomial; geometric multiplicity and algebraic multiplicity of an eigenvalue; characterization of endomorphisms and diagonalizable matrices by the existence of an eigenvector basis; determination of eigenvalues and an eigenvector basis of a diagonalizable endomorphism and a diagonalizable matrix.

Euclidean (affine) spaces over a field: 1 CFU
Definition, Cartesian (affine) references and coordinates of a point, Euclidean (affine) subspaces, definition of parallelism, twisted lines, parametric and Cartesian representation of Euclidean (affine) subspaces. Study of incidence and parallelism between subspaces. Conditions of orthogonality between subspaces in dimension 2 and 3. Distance between sets of points; distance of a point from a hyperplane; study of the distance between Euclidean subspaces in dimension 2 and 3, theorem of the perpendicular common. Definition of improper bundles and proper sheaves of planes in dimension 3.

READINGS/BIBLIOGRAPHY
SEE THE TEACHER'S WEBSITE

TEACHING METHODS
The lessons will be frontal, and about a third of the lessons will be exercised.

EXAMINATION/EVALUATION CRITERIA

a) **Exam type:**

Exam type	
written and oral	X
only written	
only oral	
project discussion	
other	

In case of a written exam, questions refer to:	**Multiple choice answers**	X
	Open answers	X
	Numerical exercises	X

b) Evaluation pattern: